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SUFFICIENT DISCRETE–INTEGRAL CRITERION OF RUPTURE STRENGTH

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The behavior of the atomic structure in the vicinity of the crack tip is modeled. The loss of stability and postcritical deformation of a triatomic cell in a close-packed atomic layer in tension are studied. For macrocracks in single crystals, the concept of the generalized Burgers vector is introduced. A sufficient discrete–integral strength criterion is proposed for normal-rupture cracks in the case where the stress fields have a singular component. In accordance with Novozhilov's hybrid model, this criterion is formulated with the use of a new class of solutions that differs from solutions used in formulating the classical sufficient strength criterion. In the limiting case where the energy characteristics of the postcritical deformation of the cell can be ignored, the sufficient criterion proposed admits a limiting passage to the necessary criterion. The critical loads calculated by means of the sufficient criterion differ substantially from those determined with the use of the necessary criterion; this makes it possible to describe the Rehbinder effect.

Introduction. In strength and fracture analyses for solids, an increasing attention has been attached to approaches based on the discrete structure of the material. Novozhilov [1] considered the fracture of an ideal crystalline solid with a crack as a discrete process and suggested to estimate the strength of a brittle elastic body in the neighborhood of singular points of the stress field by averaging the stresses within the interatomic distance and comparing these with the theoretical rupture strength. Moreover, he introduced necessary and sufficient criteria of brittle strength [1]. Real crystals contain defects among which vacancies occur most frequently. Kornev [2] proposed discrete–integral criteria for three simplest types of crack (necessary criteria according to Novozhilov's terminology). A similar approach was developed also for the complex stress–strain state under proportional loading [3] with the averaging limits for stresses being dependent on the presence, sizes, and location of defects in the neighborhood of the crack tip. Kornev et al. [4, 5] used Novozhilov's approach to obtain sufficient criteria for normal-rupture cracks. It was shown that if the crack opening is determined with the use of real potentials of interatomic interaction for an atomic chain, the theoretical strength of a crystalline solid does not depend on a concrete crystal structure in the vicinity of the crack tip.

For the necessary criteria, the corresponding averaged stresses do not exceed the theoretical rupture or shear strengths. If the necessary criterion is satisfied, the crystal structure that is the nearest to the crack tip is in a critical state. Nevertheless, after the carrying capacity of the crystal structure nearest to the tip is exhausted, additional loading of the cracked body is still possible owing to postcritical deformation of this structure and subcritical deformation of the next crystal structure provided there are no vacancies and impurity atoms in the neighborhood of the crack tip. Satisfaction of the sufficient criterion corresponds to catastrophic fracture of the system.

Let us consider the classical sufficient criteria [6–9] in greater detail. Within the framework of the linear continual model, the stresses on the continuation of a sharp crack for $y = 0$ can be written with accuracy to higher-order terms in the form

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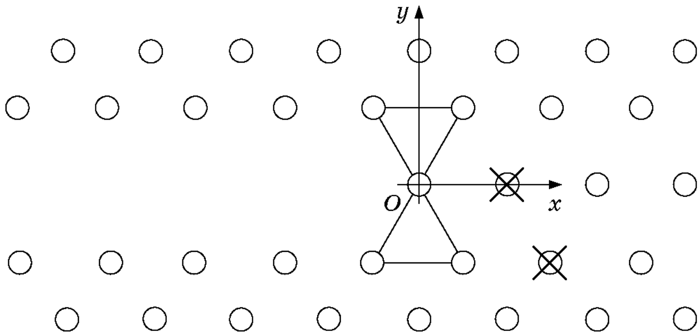

Fig. 1

$$\sigma_y(x,0) \simeq \sigma_\infty + \frac{K_{\rm I}^0}{(2\pi x)^{1/2}}, \tag{1}$$

where $K_{\rm I}^0$ is the stress-intensity factor (SIF) and σ_∞ is the characteristic stress specified at infinity or on the contour of a bounded body. It is of interest to consider two cases:

$$K_{\rm I}^0 = 0; \tag{2}$$

$$K_{\rm I}^0 > 0. \tag{3}$$

In fact, the constraint (2) is used in the classical criteria [6, 7]. Within the framework the Leonov–Panasyuk–Dugdale model, the crack forms a peculiar "nose," and the model-crack profile has an inflection point at which the tangent to the crack sides is vertical. The constraint (2) is meaningful only in the case of developed plasticity [9]. It should be noted that if the constraint (2) is satisfied, any crystal structure become unstable when the geometry of the crack sides in the neighborhood of inflection points changes. Chernykh [10] considered in detail Novozhilov's concept for the case where the constraint (2) is satisfied.

Below, we study the constraint (3). The authors consider that in this case, it is convenient to use Novozhilov's approach [10].

1. Mechanical Models and a Sufficient Criterion for Normal-Rupture Cracks. We study the behavior of a loaded body with an internal macrocrack. Let the plane macrocrack with a rectilinear front do not disrupt the monocrystal structure in the macrovolume [11]. A close-packed atomic layer that contains a macrocrack and vacancies is considered (Fig. 1). It is assumed that the macrocrack is formed in such a manner that some atoms are removed from the chain and there are vacancies in front of the tip, which are indicated by crosses in Fig. 1. Figure 2 shows the loading scheme for a triatomic cell and the typical deformation curve for the triatomic cell in tension: $\sigma_m = \max \sigma(v)$ for normal stresses; $\boldsymbol{F}$ is the force vector, $f = |\boldsymbol{F}|$, and $f_m = \max f$ (given the forces, one can determine the stresses by averaging with the use of the hybrid model in fracture mechanics), v is the displacement along the Oy axis, v_m is the displacement corresponding to f_m, and v_c is the radius of the interatomic-interaction region for the triatomic cell which is calculated from energy considerations by the rule

$$\int\limits_{v_m}^{\infty} f(v)\,dv = (v_c - v_m) f_m \tag{4}$$

if the deformation curve of the triatomic cell in tension $f = f(v)$ is known (in Fig. 2, the shaded region under the curve is equal to the area of the shaded rectangle). When the real physical potentials of interatomic interaction are used, the improper integral of the first kind in (4) converges, since the function $f(v)$ decreases rapidly as $v \to \infty$.

We model the crack by a bilateral cut. Figure 3 shows the model of a normal-rupture crack (Δ is the loaded segment of the cut) and the scheme of crack opening. For the coordinate $x = -\Delta$, the crack opening is equal to V. The possible instabilities of the atomic structures in the neighborhood of the point $x = -\Delta$ are beyond the present work.

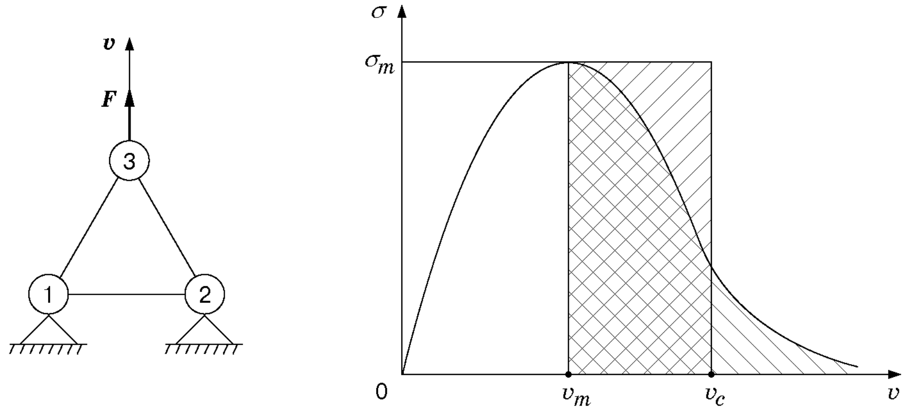

Fig. 2

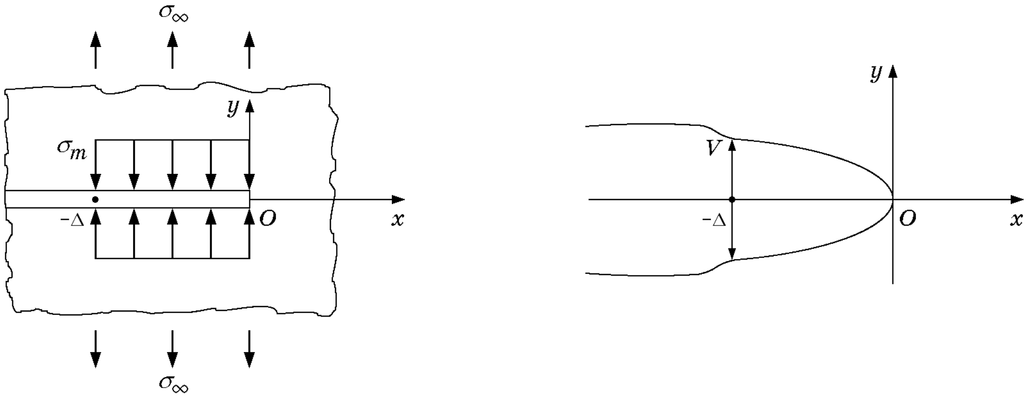

Fig. 3

We consider the weakest atomic layer located normally to the rectilinear front of a plane sharp macrocrack of length $2l$. We propose a sufficient discrete–integral criterion of quasibrittle strength for normal-rupture cracks:

$$\frac{1}{kr_e}\int_0^{nr_e}\sigma_y(x,0)\,dx \leqslant \sigma_m, \qquad x \geqslant 0; \qquad V = \frac{æ+1}{G}\,K_{\rm I}^0\sqrt{\frac{\Delta}{2\pi}} \leqslant V^*, \quad x \leqslant 0. \tag{5}$$

Here $\sigma_y(x,0)$ are the normal stresses at the crack tip in the continual model, which have an integrable singularity, Oxy is the rectangular coordinate system with the origin at the right tip of the crack, r_e is the interatomic distance, n and k are the numbers such that $n \geqslant k$ (k is the number of interatomic bonds), nr_e is the averaging interval, σ_m is the theoretical rupture strength of solids [12], V are the twice displacements of the crack sides, $V^* = v_c - v_m$ is the critical opening of the normal-rupture crack, $æ = 3-4\nu$ and $æ = (3-\nu)/(1+\nu)$ for the plane strain and the plane stress, respectively, ν is Poisson's ratio, and G is the shear modulus. After averaging with allowance for damage of the material, the stresses σ_y in the continual model are compared with the theoretical strength of ideal crystals σ_m in the discrete model. To determine the length of the loaded segment of the cut Δ that enters the sufficient criterion (5), one can use either the concrete crystal structure of the material in the vicinity of the crack tip or the real physical potentials of interatomic interaction. The crack sides interact only on the loaded segment of the cut.

It is obvious that $V \to 0$ as $\Delta \to 0$ and the sufficient criterion (5) becomes a necessary criterion [2, 3]. The averaging limits for stresses in the necessary and sufficient criteria depend on the presence, sizes, and location of defects in the crystal lattice near the crack tip. For the close-packed atomic layer shown in Fig. 1, we have $n = 2$ and $k = 1$. The magnitude of these averaged stresses must be smaller than the theoretical strength σ_m. The ratio k/n characterizes the degree of damage of the continuous material in front of the crack tip. Before considering a similar parameter in the interval $[-\Delta; 0]$ for a material before rupture, one should verify whether the constraint $\Delta/r_e \geqslant 2$ is satisfied.

Let there be a sharp crack of length $2l^0_{nk}$ such that $\Delta = 0$. Under successive additional loading, the crack does not increase provided $\sigma_\infty < \sigma^0_\infty$ (σ^0_∞ are the critical stresses for sharp cracks obtained by means of the necessary criteria [2, 3] for a crack length equal to $2l^0_{nk}$). When the load exceeds the critical stresses for the necessary criterion ($\sigma_\infty > \sigma^0_\infty$), the crack begins to increase and the triatomic cells that are the nearest to the crack tip begin to "work" in the postcritical regime; at the same time, the force constraints form in the neighborhood of the crack tip, and the reference point in the model shown in Fig. 3 is shifted. Owing to force constraints, the growth of the crack is stable $2l^0_{nk} < 2l_{nk} < 2l^{*0}_{nk}$ up to a certain loading level σ^{*0}_∞ (σ^{*0}_∞ are the critical stresses determined by means of the sufficient criterion for sharp cracks of length l^{*0}_{nk}). Under gradual loading such that $\sigma^0_\infty < \sigma_\infty < \sigma^{*0}_\infty$, the crack opening V increases ($V < V^*$) and the growth of the crack is stable: the loads increase with the length of the crack: $\Delta < \Delta^*$ (Δ^* is the critical length of the loaded segment of the cut). When the length of the loaded segment of the cut Δ reaches the critical value Δ^*, i.e., $V = V^*$, the growth of the crack becomes unstable. As a result, a peculiar trap forms for cracks propagating in quasibrittle materials.

We obtain relations between the critical parameters $K^{*0}_{\rm I}$ and Δ^* for sharp cracks. After appropriate transformations, we have

$$\frac{K^{*0}_{\rm I}}{\sigma^{*0}_\infty\sqrt{r_e}} = \sqrt{\frac{\pi}{2}}\, n\Big(\frac{\sigma_m}{\sigma^{*0}_\infty}\,\frac{k}{n} - 1\Big), \qquad \Delta^* = 2\pi\Big(\frac{G}{æ+1}\,\frac{V^*}{K^{*0}_{\rm I}}\Big)^2. \tag{6}$$

The first relation in (6) coincides with the critical SIF that enters the necessary criterion of brittle strength [2] with accuracy to notation.

In accordance with the model proposed, the SIF $K^0_{\rm I}$ of the sufficient criterion is written in the form

$$K^0_{\rm I} = K^0_{{\rm I}\infty} + K^0_{{\rm I}\Delta}, \tag{7}$$

where $K^0_{{\rm I}\infty}$ is the SIF generated by the stresses σ_∞ and $K^0_{{\rm I}\Delta}$ is the SIF generated by the stresses σ_m which act in the vicinity of the crack tip. We recall that, in accordance with the model of a normal-rupture crack (Fig. 3), the SIF $K^0_{{\rm I}\infty}$ is expressed in terms of the stresses σ_∞ at infinity and the half-length of the internal crack l_{nk}, whereas the SIF $K^0_{{\rm I}\Delta}$ is expressed in terms of the stresses σ_m, the half-length of the crack l_{nk}, and the length of the loaded segment of the cut Δ as follows: $K^0_{{\rm I}\infty} = \sigma_\infty\sqrt{\pi l_{nk}}$ and $K^0_{{\rm I}\Delta} = -\sigma_m\sqrt{\pi l_{nk}}(1 - (2/\pi)\arcsin(1 - \Delta/l_{nk}))$. The stresses σ_∞ determine the smooth part of the solution in the vicinity of the crack tip [see (1)], whereas the stresses σ_m specified on the opposite sides of the cut (crack) are self-balanced stresses and do not determine it. Taking into account the direction of action of the tensile stresses at infinity σ_∞ and the compressive stresses σ_m specified in the interval $[-\Delta; 0]$, in accordance with the sufficient criterion, we finally obtain $K^0_{\rm I}$ for the internal normal-rupture cracks [see (7)]:

$$K^0_{\rm I} = \sigma_\infty\sqrt{\pi l_{nk}} - \sigma_m\sqrt{\pi l_{nk}}[1 - (2/\pi)\arcsin(1 - \Delta/l_{nk})] > 0. \tag{8}$$

In formula (8), we have $0 \leqslant \Delta \leqslant \Delta^*$ and $2l^0_{nk} \leqslant 2l_{nk} \leqslant 2l^{*0}_{nk}$. Obviously, for $\Delta = 0$, the SIF determined by means of the sufficient criterion is equal to the SIF determined by means of the necessary criterion, since $K^0_{{\rm I}\Delta} = 0$ in this case.

We now estimate the quantity Δ. Relation (8) can be simplified significantly if the length of the loaded segment $[-\Delta; 0]$ is much smaller than the half-length of the crack, i.e., $\Delta/l_{nk} \ll 1$. In this case, we obtain $\arcsin(1 - \Delta/l_{nk}) \simeq \pi/2 - \sqrt{2\Delta/l_{nk}}$. Using the sufficient criterion (5), after certain manipulations we obtain the quadratic equations for the dimensionless parameter $\sqrt{\Delta/l_{nk}}$

$$\Big(\sqrt{\frac{\Delta^*}{l^{*0}_{nk}}}\Big)^2 - \frac{\pi}{2\sqrt{2}}\,\frac{\sigma^{*0}_\infty}{\sigma_m}\sqrt{\frac{\Delta^*}{l^{*0}_{nk}}} + \frac{\pi}{2(æ+1)}\,\frac{V^*}{l^{*0}_{nk}}\,\frac{G}{\sigma_m} = 0.$$

Neglecting the quantity Δ^*/l^{*0}_{nk} compared to unity, we obtain a simpler expression for the smaller root of the quadratic equation

$$\sqrt{\frac{\Delta^*}{l^{*0}_{nk}}} \simeq \frac{\sqrt{2}}{æ+1}\,\frac{V^*}{l^{*0}_{nk}}\,\frac{G}{\sigma^{*0}_\infty}. \tag{9}$$

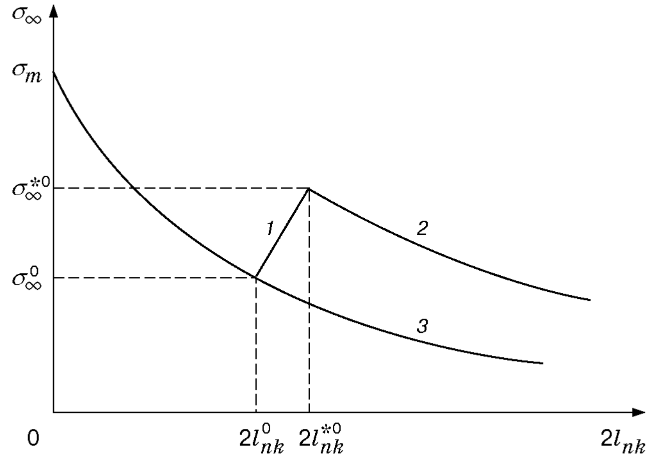

Fig. 4

Remark 1. If the constraint $\Delta^*/l_{nk}^{*0} \ll 1$ is not satisfied, the sufficient criterion (5) or relation (7) yields a transcendental equation for Δ^*/l_{nk}^{*0}. No serious difficulties arise in solving this equation if its positive roots are smaller than unity.

Substituting (9) into (8), we obtain the critical SIF $K_{\rm I}^{*0}$ for a sharp internal normal-rupture crack for $\Delta^*/l_{nk}^{*0} \ll 1$:

$$\frac{K_{\rm I}^{*0}}{\sigma_\infty^{*0}\sqrt{\pi l_{nk}^{*0}}} = 1 - \frac{\sigma_m}{\sigma_\infty^{*0}}\,\frac{2\sqrt{2}}{\pi}\,\sqrt{\frac{\Delta^*}{l_{nk}^{*0}}}. \tag{10}$$

Thus, for a given load, the rupture curve obtained by means of the sufficient criterion and the critical length of a sharp internal normal-rupture crack are given by

$$\frac{\sigma_\infty^{*0}}{\sigma_m} = \left(\frac{n}{k} + \frac{\sqrt{n}}{k}\sqrt{\frac{2l_{nk}^{*0}}{r_e}}\right)^{-1}\left(1 + \frac{4\sqrt{n}}{\pi k}\sqrt{\frac{\Delta^*}{r_e}}\right); \tag{11}$$

$$\frac{2l_{nk}^{*0}}{r_e} = \left[\frac{\sigma_m}{\sigma_\infty^{*0}}\left(1 + \frac{4\sqrt{n}}{\pi k}\sqrt{\frac{\Delta^*}{r_e}}\right) - \frac{n}{k}\right]\frac{k^2}{n}. \tag{12}$$

The equations of rupture curves obtained by means of the sufficient criterion (11) differ from those obtained by means of the necessary criterion [2, 3] by the last factor which depends on the length of the loaded segment of the cut. Relations (10)–(12) admit a limiting passage if the SIF, the length of the loaded segment of the cut, and the length of the crack tend to zero.

We compare the critical loads obtained with the use of the necessary and sufficient criteria for brittle materials for the same lengths of the cracks:

$$\frac{\sigma_\infty^{*0}}{\sigma_\infty^0} = 1 + \frac{4\sqrt{n}}{\pi k}\sqrt{\frac{\Delta^*}{r_e}}. \tag{13}$$

These critical loads differ appreciably. The discrepancy in the critical loads can be attributed to the Rehbinder effect [13–15].

Figure 4 shows schematically the stable and unstable parts of the crack growth (curves 1 and 2, respectively) and the rupture curve obtained by means of the necessary criteria [2, 3] (curve 3). On the stable part, the formed system sustain an increased load, since $\sigma_\infty^{*0} > \sigma_\infty^0$; as a result, the crack extends, since $l_{nk}^0 < l_{nk}^{*0}$.

2. Estimated Strength of a Triatomic Cell. The behavior of the atomic structure in the vicinity of the crack tip is modeled. The instability of a triatomic cell in a close-packed atomic layer in tension is studied. We consider deformation of the triatomic cell shown in Fig. 2. The external action is characterized

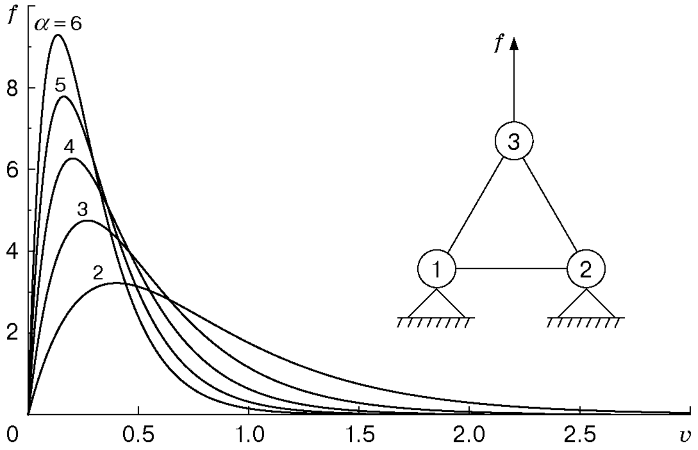

Fig. 5

TABLE 1

α	v_m/r_e	V^*/r_e	α	v_m/r_e	V^*/r_e
2	0.400	0.791	7	0.115	0.239
3	0.269	0.537	8	0.101	0.209
4	0.202	0.408	9	0.090	0.187
5	0.162	0.329	10	0.081	0.168
6	0.135	0.276			

by the force $\boldsymbol{F}$ applied to the third atom of the cell. The action of the interatomic forces is assumed to be central with the Morse potential of interaction [12]

$$U(r) = D[\mathrm{e}^{-2\alpha(r-r_e)} - 2\,\mathrm{e}^{-\alpha(r-r_e)}],$$

where r is the interatomic distance, r_e is the interatomic distance in the state of equilibrium, and D and α are constants. For $r = r_e$ (state of equilibrium), the central force of interatomic interaction vanishes; the repulsive (negative) force acts between the atoms for $r < r_e$, and attraction (positive) force for $r > r_e$, the latter reaches the maximum f_m at a certain distance r_m and then decreases with an increased interatomic distance and becomes an order of magnitude smaller than its maximum at the distance $2r_e$. The first derivative of the Morse potential yields the central force

$$f(r) = \frac{\partial U(r)}{\partial r} = 2D\alpha[\mathrm{e}^{-\alpha(r-r_e)} - \mathrm{e}^{-2\alpha(r-r_e)}],$$

whence $r_m = r_e + \ln(2/\alpha)$ and $f_m = D\alpha/2$.

The nonlinear problem of deformation of the atomic cell is solved by the finite-element method [16]. The triatomic cell is treated as a pivotal structure in which nodes 1 and 2 are fixed and node 3 has two degrees of freedom. Under the action of external forces, the structure undergoes extension. The problem of deformation of an atomic cell was solved numerically with the use of a step-by-step technique [17]. Because of large displacements and rotations, the physically nonlinear problem of deformation of an atomic cell is also a geometrically nonlinear problem. The solutions of similar problems contain eigenstates of the maximum-load type. The main difficulty in solving these problems is that the external load which acts on the crystal lattice cannot be used as a monotonically increasing deformation parameter. Moreover, the tangent stiffness matrix degenerates when the load reaches its maximum. In this case, the Newton–Raphson iterative procedure does not yield the convergence to the solution of the problem. To overcome these difficulties, Korobeinikov [17] suggested to consider the external-load parameter as a desired quantity and specify the arc length in the $(\boldsymbol{U}, \lambda)$ space ($\boldsymbol{U}$ is the displacement vector and λ is the external-load parameter) as an additional equation.

Numerical calculations were performed for the following dimensionless constants of the interatomic-interaction potential: $r_e = 1$, $D = 1.7$, and $\alpha = 2$–10. The radius of the interatomic-interaction region v_c was determined from formula (4). Figure 5 shows the force versus the displacement for a triatomic cell in tension. Table 1 lists the calculated values of v_m and V^* for certain values of the parameter α in tension. We note that the values of V^* do not depend on the parameter D. The force–displacement relations obtained support the hypothesis proposed in Sec. 1: in formula (4), $f(v) > 0$ and $f(v) \to 0$ as $v \to \infty$.

Using the values of V^*/r_e obtained and relations (9) and (13), one can calculate the dimensionless lengths of the loaded segments of cracks Δ^*/r_e and the ratio of the critical loads $\sigma_\infty^{*0}/\sigma_\infty^0$. The numerical calculations show that in some cases, one can obtain simple estimates. To this end, we write formula (9) in the form

$$\sqrt{\frac{\Delta^*}{r_e}} \simeq \frac{\sqrt{2}}{æ+1}\,\frac{V^*}{r_e}\,\sqrt{\frac{r_e}{l_{nk}^{*0}}}\,\frac{G}{\sigma_\infty^{*0}}. \tag{14}$$

The approximate equality (14) includes the quantities σ_∞^{*0} and l_{nk}^{*0}. To obtain estimates, we use the following relations: 1) if the length determined by means of the necessary and sufficient criteria coincide, the critical loads are subject to the condition $\sigma_\infty^{*0}(l_{nk}^0) > \sigma_\infty^0(l_{nk}^0)$ with $\sigma_\infty^{*0} \to \sigma_\infty^0$ as $\Delta \to 0$; 2) if the loads are such that $\sigma_\infty^0 < \sigma_\infty < \sigma_\infty^{*0}$, we have $2l_{nk}^0 < 2l_{nk} < 2l_{nk}^{*0}$, since $l_{nk}^{*0} = l_{nk}^0 + \Delta^*$. Replacing the quantities σ_∞^{*0} and l_{nk}^{*0} in (14) by the quantities σ_∞^0 and l_{nk}^0, respectively, we obtain the approximate inequality

$$\sqrt{\frac{\Delta^*}{r_e}} \lesssim \frac{\sqrt{2}}{æ+1}\,\frac{V^*}{r_e}\,\sqrt{\frac{r_e}{l_{nk}^{0}}}\,\frac{G}{\sigma_\infty^{0}}. \tag{15}$$

Using the necessary criterion, Kornev and Kurguzov [2, 3] obtained the relation for the critical stress σ_∞^0 (rupture curves) for a sharp internal crack

$$\frac{\sigma_\infty^0}{\sigma_m} = \left(\frac{n}{k} + \frac{\sqrt{n}}{k}\sqrt{\frac{2l_{nk}^0}{r_e}}\right)^{-1}.$$

This relation is simplified for reasonably long cracks if $2l_{nk}^0/r_e \gg 1$:

$$\frac{\sigma_\infty^0}{\sigma_m} \simeq \frac{k}{\sqrt{n}}\sqrt{\frac{r_e}{2l_{nk}^0}}. \tag{16}$$

According to [12], estimates of the theoretical strength have the form

$$\sigma_m = \eta_1 E, \tag{17}$$

where $0.1 < \eta_1 < 0.3$. Substituting (16) and (17) into (15), we finally obtain

$$\sqrt{\frac{\Delta^*}{r_e}} \lesssim \frac{1}{æ+1}\,\frac{V^*}{r_e}\,\frac{\sqrt{n}}{k(1+\nu)\eta_1}. \tag{18}$$

For the same lengths of long cracks, the estimates of critical loads obtained by means of the necessary and sufficient criteria for brittle materials have the form

$$\frac{\sigma_\infty^{*0}}{\sigma_\infty^0} \lesssim 1 + \frac{4n}{\pi k^2(1+\nu)\eta_1}\,\frac{1}{æ+1}\,\frac{V^*}{r_e}. \tag{19}$$

Table 2 lists calculation results obtained with the use of (18) and (19) for plane strain (massive bodies) and plane stress (thin films deposited on a flexible substrate).

For real values of α ($3 \leqslant \alpha \leqslant 6$), the ratio $\Delta^*/r_e = 1.1$–0.23. For these lengths of the loaded segments, it makes no sense to consider the damage of the material with the use of sufficient criteria. Tables 1 and 2 give the dependence of critical loads obtained by means of the sufficient criterion on the energy characteristics of the postcritical deformation of triatomic cells.

3. The Generalized Burgers Vector. In contrast to the formulation of necessary criteria in [2, 3], the main additional element in the formulation of the sufficient criterion (5) is the critical opening of a normal-rupture crack V^*. Following Cottrell [18], we give two equivalent definitions of the Burgers vector.

TABLE 2

α	Plane strain		Plane stress	
	Δ^*/r_e	$\sigma_\infty^{*0}/\sigma_\infty^0$	Δ^*/r_e	$\sigma_\infty^{*0}/\sigma_\infty^0$
2	2.369	3.769	1.955	3.519
3	1.092	2.880	0.901	2.710
4	0.630	2.428	0.520	2.299
6	0.288	1.966	0.238	1.879
8	0.165	1.732	0.137	1.666
10	0.107	1.588	0.088	1.535

Definition 1 (with the use of the elastic field)**.** Let there be a macrocrack under specified loading. We choose a sufficiently large closed contour in a continuous material that intersects the macrocrack at the point $(-\Delta; 0)$. The Burgers contour is traced counter-clockwise. Let ds be the element of the Burgers vector and v be the displacement along the y axis. Then, the opening of the loaded normal-rupture crack V is expressed in terms of the contour integral

$$V = \oint \frac{\partial v}{\partial s}\, ds. \tag{20}$$

Relation (20) determines the components of the generalized Burgers vector $\boldsymbol{b} = (0, V)$.

Definition 2 (of the Burgers–Frank type)**.** Let there be a macrocrack under specified loading. We consider a closed contour in the ideal crystal lattice (there is no macrocrack). Let this contour pass through the point $(-\Delta; 0)$. Then, the corresponding contour that encloses the macrocrack tip is open and the generalized Burgers vector is the vector $\boldsymbol{b}$ of an ideal lattice which corresponds to the discontinuity of the contour in the defect crystal after a macrocrack is introduced.

The definitions proposed, which differ from those accepted in the physics of solids, can be used to formulate a sufficient strength criterion for the generalized stress state [3]. In this case, the generalized Burgers vector has two components: $\boldsymbol{b} = (U, V)$.

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